Matching search frictions

Alfred Galichon
"Matching Market" course
Sciences Po

April, 16 2013

1 Introduction

The modelling carried thus far is static in the sense that it assumes that agents have a technology to meet the other side of the market all at once, and that negociation is free and immediate. It also assumes that all the matches are formed at the same time.

In fact , such is not the case. Agents do not have access to the totality of the market all at once, but they only have access to a limited sample drawn randomly from the population. In one extreme case, one may assume that agents meet one member of the other side of the market at once, and make a decision whether to form a match or not. The presence of search frictions will induce several modifications:

1. Remaining unmatched spares the agent the possibility to match at a future date: thus intertemporal utility

of unmatched agents will no longer be zero, but will be endogenously determined as an option value.

2. As waiting is coslty to agents, matches which would be found suboptimal in the frictionless limit may become acceptable by agents. Hence the set of possible matches will be expanded.

3. As agents do not have access to the totality of the market at once, there is more room for bargaining due do decreased competitive pressure: agents will bargain over how to share the surplus with respect to status quo utility (=option value of remaining single).

In this lecture, we shall study the existence of equilibria in such settings. We shall be interested in the qualitative properties of equilibria, in particular under the assumption that the production function satisfies Increasing Differences, whether the intuition that "high types match with high types" carries over. We shall see that it may not. Indeed, take $\Phi(x,y) = (x+y-1)^2$ as suggested by Shimer and Smith (200), p and q uniform over $[0,1]$. In the static case, $y = T(x)$; but in the case with search fritions, when for instance type $1/2$ and meets type $1/2$, they choose not to meet: indeed they would get zero out of this matching, while if they wait, they are likely to get a nonzero payoff.

We shall review two models. One, by Shimer and Smith, is a search equilibrium with time discounting and destruction and reformation of matches. The other models, by Atakan (2006), assumes additive search costs, no destruction of matches and replacement of matched individuals by their identical unmatched clones. It has the drawback of requiring less convincing assumptions, but the derivation of the model is strikingly simple as we shall see.

1.1 Frictions with time discounting

In a seminal paper, Shimer and Smith (2000) show that when agents meet randomly and when wating is coslty, the intuition (true with a continuum of agent under supermodular surplus) that the optimal matching is one-to-one no longer holds. Instead, the matching assignment map $T(x)$ will be replaced by a *matching correspondence* $A(x)$, which is the set of types of firms CEO x may be willing to match with. Time is too costly to wait for the optimal match, which will not arrive before an infinite amount of time anyway, so the CEO lowers her standards, and is willing to settle for matches that are close to optimal. When the matches are too bad, the CEO may prefer waiting for a better match.

1.1.1 Search Equilibrium

The setting is as before: there are distributions (non necessarily probabilities) $p(x)$ and $q(y)$ over the type spaces of employees and firms, respectively. It is assumed that agents x and y produce together a flow $\phi(x, y)$ of output per unit of time; an unmatched agent produces 0.

Some assumptions are made on the production function ϕ.

A0. Assume $\mathcal{X} = \mathcal{Y} = [0, 1]$ and $\phi(x, y) \geq 0$ is C^2.

A1. ϕ is strictly supermodular.

At each period, the surplus of a matched pair is split in the following way

$$\phi(x, y) = u(x, y) + v(x, y).$$

A given matched pair (x, y) remains matched under the surplus sharing conditions given by this surplus sharing rule until the match is destroyed, which occurs according to a Poisson process of intensity δ. Unmatched agents match randomly according to a Poisson process of intensity ρ.

Agents discount future earnings with exponential discount factor r. Let $U_0(x)$ and $V_0(y)$ be the intertemporal value of unmatched agents x and y, and let

$$\begin{aligned} \Delta U(x,y) &= U(x,y) - U_0(x) \\ \Delta V(x,y) &= V(x,y) - V_0(y). \end{aligned}$$

Let A be the set of mutually acceptable matches, ie.

$$A = \{(x,y) \in \mathcal{X} \times \mathcal{Y} : \Delta U(x,y) \geq 0,\ \Delta V(x,y) \geq 0\}$$

As a notational shorthand, one shall denote $y \in A(x)$, $x \in A(y)$.

The intertemporal utility of unmatched agents results from the expectation of meeting an acceptable match. Therefore, the Bellman equation of unmatched workers and firms is given by

$$
\begin{aligned}
rU_0(x) &= \rho \int_{A(x)} \Delta U(x,y)\, d\mu_0(y) && (1.1)\\
&= : u_0(x) \\
rV_0(y) &= \rho \int_{N(y)} \Delta V(x,y)\, d\mu_0(x) && (1.2)\\
&= : v_0(y).
\end{aligned}
$$

where $d\mu_0(x)$ and $d\mu_0(y)$ denote the measures of unmatched workers of type x and y.

Let us work out the expression of the steady state matching $\mu(x,y)$; a flow $\delta\mu(x,y)$ is exogenously destroyed, while a flow $\rho\mu_0(x)\mu_0(y) 1\{(x,y)\in A\}$ is exogenously created. As steady state, one has the matching flow equation

$$\delta\mu(x,y) = \rho\mu_0(x)\mu_0(y) 1\{(x,y)\in A\} \qquad (1.3)$$

thus, integrating over y

$$
\begin{aligned}
\delta(p(x)-\mu_0(x)) &= \rho\mu_0(x)\int 1\{(x,y)\in A\}\, d\mu_0(y) && (1.4)\\
\delta(q(y)-\mu_0(y)) &= \rho\mu_0(y)\int 1\{(x,y)\in A\}\, d\mu_0(x) && (1.5)
\end{aligned}
$$

For matched workers, the intertemporal utility results from the flow of payoffs and the risk of the destruction of the match. Hence

$$\begin{aligned} rU(x,y) &= u(x,y) - \delta \Delta U(x,y) \\ rV(x,y) &= v(x,y) - \delta \Delta V(x,y) \end{aligned}$$

rearranging these equations yields

$$\begin{aligned} (r+\delta)\,\Delta U(x,y) &= u(x,y) - u_0(x) \\ (r+\delta)\,\Delta V(x,y) &= v(x,y) - v_0(y) \end{aligned}$$

Note that even with a continuum of types x and y, it is not clear how the surplus is shared: indeed, the solution cannot rely on competitive equilibrium because the scarsity of encounters drastically limits competition. We have to assume there is a form of bargaining between x and y in order to share the pie $\phi(x,y)$. We shall assume the Nash bargaining solution, which implies that $u(x,y) - u_0(x) = v(x,y) - v_0(y)$, thus

$$\Delta U(x,y) = \Delta V(x,y) = \frac{\phi(x,y) - u_0(x) - v_0(y)}{2(r+\delta)}.$$

As a result, the set of mutually acceptable matches is given by

$$A = \{(x,y) \in \mathcal{X} \times \mathcal{Y} : \phi(x,y) - u_0(x) - v_0(y) \geq 0\}.$$

Also, plugging in into the Bellman equations (1.1) and (1.2), one gets

$$\begin{aligned} u_0(x) &= \theta \int_{A(x)} \phi(x,y) - u_0(x) - v_0(y)\, d\mu_0(y) \\ v_0(y) &= \theta \int_{A(y)} \phi(x,y) - u_0(x) - v_0(y)\, d\mu_0(x) \end{aligned}$$

where

$$\theta = \frac{\rho}{2(r+\delta)}$$

An equilibrium can therefore be stated in terms of the functions u_0 and v_0 and the densities of unmatched individuals $\mu_0(x)$ and $\mu_0(y)$. A Search Equilibrium (SE) is a quadruple $(u_0(x), v_0(y), \mu_0(x), \mu_0(y))$ such that

$$\begin{aligned} u_0(x) &= \theta \int_{A(x)} \phi(x,y) - u_0(x) - v_0(y)\, d\mu_0(y) \\ v_0(y) &= \theta \int_{A(y)} \phi(x,y) - u_0(x) - v_0(y)\, d\mu_0(x) \end{aligned}$$

and

$$\begin{aligned} \delta(p(x) - \mu_0(x)) &= \rho \mu_0(x) \int_{A(x)} d\mu_0(y) \\ \delta(q(y) - \mu_0(y)) &= \rho \mu_0(y) \int_{A(y)} d\mu_0(x). \end{aligned}$$

Under assumptions (A0) and (A1), one has the existence of a SE.

Theorem 1.1 (Shimer and Smith)**.** *Under Assumptions (A0) and (A1), there exists a SE.*

Sketch of the proof. The idea is to reformulate the problem as a fixed point problem. We are looking for u_0 and v_0 such that

$$u_0(x) = \theta \int (\phi(x,y) - u_0(x) - v_0(y))^+ \, d\mu_0(y)$$

and a similar equation for v_0, that is, we are looking for a fixed point of operator T defined as

$$\begin{aligned} T^1(u_0, v_0)(x) &= \frac{\theta \int \max(\phi(x,y) - v_0(y), u_0(x))\, d\mu_0(y)}{1 + \theta \int_{A(x)} d\mu_0(y)} \\ T^2(u_0, v_0)(x) &= \frac{\theta \int \max(\phi(x,y) - u_0(x), v_0(y))\, d\mu_0(x)}{1 + \theta \int_{A(y)} d\mu_0(x)} \end{aligned}$$

where $\mu_0(dx)$ and $\mu_0(dy)$ are implicitely defined from u_0 and v_0 by

$$\begin{aligned} \delta(p(x) - \mu_0(x)) &= \rho\mu_0(x) \int_{A(x)} d\mu_0(y) \\ \delta(q(y) - \mu_0(y)) &= \rho\mu_0(y) \int_{A(y)} d\mu_0(x). \end{aligned}$$

It can be shown ([**?**], Lemmas 3 and 4) that the map $(u_0, v_0) \to (\mu_0(dx), \mu_0(dy))$ is well-defined and continuous.

The existence of a fixed point is established by appeal to the Schauder Fixed Point Theorem; see [**?**], Proposition 1 for detail. ■

Proposition 1.1. *Under the assumption (A0), for given distributions* $\mu_0(dx)$ *and* $\mu_0(dy)$ *of unmatched individuals(ii)* $u_0(x)$ *satisfies, the following holds:*

(i) for every $M \subset [0,1]$

$$\begin{aligned} u_0(x) &\geq \theta \int_M (\phi(x,y) - u_0(x) - v_0(y))\, d\mu_0(y) \\ v_0(y) &\geq \theta \int_M (\phi(x,y) - u_0(x) - v_0(y))\, d\mu_0(x). \end{aligned}$$

(ii) when the involved quantities exists,

$$u_0'(x) = \frac{\theta \int_{A(x)} \partial_x \phi(x,y)\, d\mu_0(y)}{1 + \theta \int_{A(x)} d\mu_0(y)} \qquad (1.6)$$

and a similar equation holds for $v_0'(y)$.

Proof. (i) follows from the fact that $u_0(x) = \int_{A(x)} \phi(x,y) - u_0(x) - v_0(y)\, d\mu_0(y)$ where

$$A(x) = \{y : \phi(x,y) - u_0(x) - v_0(y) \geq 0\}.$$

Part (ii) follows from the enveloppe theorem, which yields

$$u_0'(x) = \theta \int_{A(x)} \partial_x \phi(x,y) - u_0'(x)\, d\mu_0(y).$$

■

Note that the matching flow equation (1.3) rewrites as

$$\frac{\mu(x,y)}{\mu_0(x)\,\mu_0(y)} = \frac{\rho}{\delta} 1\{(x,y) \in A\}$$

hence the scaled number of matched pairs identifies A and $\frac{\rho}{\delta}$ in this model. This is somehow reminiscent of Choo and Siow's identifying equation; however, the normalization is by $\mu_0(x)\,\mu_0(y)$ unlike in Choo and Siow's model where it is by $\sqrt{\mu_0(x)\,\mu_0(y)}$. Of course, the set of acceptable matchings A is not a primitive of the model.

1.1.2 Generalized Positive Assortative Matching

Shimer and Smith look for regularity conditions under which the intuitions of the positive assortative matching generalize to SE. In the model witout frictions, recall that $A(x) = \{T(x)\}$ is a single point, where $T(x) = F_q^{-1} \circ F_p(x)$.

In the current setting, it would be natural to expect that

$$A(x) = \left[\underline{T}(x), \overline{T}(x)\right]$$

where $\underline{T}$ and $\overline{T}$ are nondecreasing.

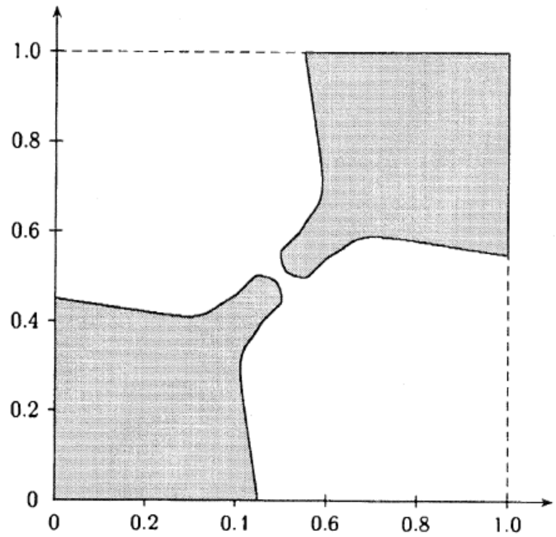
However, this intuition fails, as shown in the examples of Figure (1): more conditions are needed.

Introduce the following conditions:

(A2) $\log \frac{\partial \phi}{\partial x}(x, y)$ and $\log \frac{\partial \phi}{\partial y}(x, y)$ are supermodular

(A3) $\log \frac{\partial^2 \phi}{\partial x \partial y}(x, y)$ is supermodular.

Then:



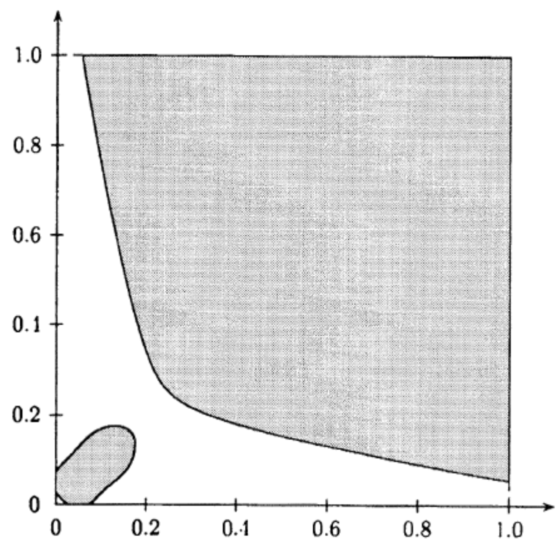


FIGURE 3.—Non-Convex Matching. The top panel depicts matching sets for $f(x, y) = (x + y - 1)^2$, $\delta = r$, $\rho = 100r$, and $L(x) = x$ on $[0, 1]$. The bottom panel depicts matching sets for $f(x, y) = (x + y)^2$, $\delta = r$, $\rho = 35r$, and $L(x) = x$ on $[0, 1]$.

Figure 1: Source: Shimer and Smith (2000).

Theorem 1.2 (Shimer and Smith). *Under assumptions (A0), (A1), (A2) and (A3), and assuming*

$$\frac{\partial \phi}{\partial y}(0,y) \leq 0 \leq \frac{\partial \phi}{\partial y}(1,y) \text{ for all } y \qquad (1.7)$$

one has

$$A(x) = \left[\underline{T}(x), \overline{T}(x)\right]$$

where $\underline{T}$ *and* $\overline{T}$ *are nondecreasing.*

The proof of this result proof is rather technical, and will be omitted; refer to [**?**], Proposition 6 and Lemma 5. It consists in showing that under (A0), (A1), (A2) and (A3), then the set A of acceptable matches is convex. This is a direct consequence of the fact hat under the assumptions made, ϕ is quasiconcave in both its arguments. Next, the lower and upper boundaries of $A(x)$ are shown to be nondecreasing as a consequence of the further assumption (1.7).

1.1.3 Frictionless limit

Let us now describe intuitively what happens when the model tends to the Becker model. In this case:

- agents are infinitely patient: $r \to 0$,
- matches are destroyed at a finite rate: $\delta = 1$
- agents meet with an infinite intensity rate: $\rho \to +\infty$

Then $\theta = \frac{\rho}{2(r+\delta)} \to +\infty$. In this case, Equation (1.6) becomes

$$u_0'(x) = \frac{\int_{A(x)} \partial_x \phi(x,y)\, d\mu_0(y)}{\int_{A(x)} d\mu_0(y)} \tag{1.8}$$

and as $\left[\underline{T}(x), \overline{T}(x)\right]$ will shrink to $T(x)$ so

$$u_0'(x) \to \partial_x \phi(x, T(x))$$

which is the classical compensating wages differentials equation from the model without frictions.

1.2 Frictions with additive search cost

In a subsequent paper, Atakan (2006) assumes a discrete time model where agents unmatched agents pay a per-period search cost c, until they agree to match. If x agrees to match with y at time t, then x

In this model, we assume $\mathcal{X} = \mathcal{Y} = [0, 1]$ and adopt assumptions A0 and A1 as before. At each period, agents meet a randomly selected partner, and have to decide whether to match or not. Each unmatched agent needs to pay a fixed cost at every period. Agents who matched get a payoff from the joint production and are then withdrawn from the population. It is assumed that they are replaced by their identical clones, so that, unlike what happens in the previous model, a match between agents does not affect the distributions p and q.

Let $U_0(x)$ and $V_0(y)$ be the intertemporal utilities of unmatched agent.

- Prior to matching, agents will receive negative payment $-c$ at each period (search cost).

- Agent who decide to match will get $\Phi(x,y)$ once for ever and be removed of the market. They will share the surplus using the Nash bargaining solution, hence if x and y match, then x will get payment
$$-c + \frac{\Phi(x,y) + U_0(x) - V_0(y)}{2}$$
and y will get
$$-c + \frac{\Phi(x,y) - U_0(x) + V_0(y)}{2}.$$

- After being matched, agents will receive zero payment at each future period.

Note that, compared to the previous setting, these assumptions are peculiar in the sense that they do not allow agents to opt out of the game: unmatched agents have no choice but pay c at every period. This will imply that unmatched agents' values are possibly negative, which implies that agents' reservation utilities are $-\infty$.

Agent x has utility stock $U_0(x)$ if she remains unemployed, and $\frac{\Phi(x,y)+U_0(x)-V_0(y)}{2}$ if she decides to match with y. Thus she will agree on the match with y if and only if

$$\Phi(x, y) - U_0(x) - V_0(y) \geq 0 \Leftrightarrow: (x, y) \in A$$

The Bellman equation of x is therefore

$$\begin{aligned} U_0(x) &= -c + U_0(x) \\ &\quad + \mathbb{E}_Y\left[\left(\frac{\Phi(x, Y) - U_0(x) - V_0(Y)}{2}\right)^+\right] \end{aligned}$$

which leads to the following proposition:

Theorem 1.3 (Atakan)**.** *The following holds:*

(i) The matching functions $U_0(x)$ and $V_0(y)$ verify for all x and y

$$\mathbb{E}_Y\left[(s(x,Y))^+\right] = 2c \quad (1.9)$$
$$\mathbb{E}_X\left[(s(X,y))^+\right] = 2c, \quad (1.10)$$

where

$$s(x,y) = \Phi(x,y) - U_0(x) - V_0(y).$$

(ii) Under assumptions (A0) and (A1), one has

$$A(x) = \left[\underline{T}(x), \overline{T}(x)\right]$$

where $\underline{T}$ and $\overline{T}$ are nondecreasing, and

$$\underline{T}(x) \leq F_q^{-1} \circ F_p(x) \leq \overline{T}(x).$$

Sketch of the proof. Point (i) has been established above. For (ii), we shall admit without a proof that $A(x)$ is convex. This implies that

$$A(x) = \left[\underline{T}(x), \overline{T}(x)\right]$$

and let us show that $\underline{T}(x)$ and $\overline{T}(x)$ are nondecreasing. Note that

$$(x,y) \in A \Leftrightarrow E_Z\left[(s(x,Z) - s(x,y))^+\right] \leq 2c.$$

Indeed, function

$$\varphi(x) = E_Z\left[(s(x,Z) - x)^+\right]$$

is strictly decreasing around 0 and equal to $2c$ for $x = 0$, thus $s(x,y) \geq 0$ is equivalent to $\varphi(s(x,y)) \leq 2c$.

Assume that $y < \underline{T}(x)$. Then $E_Z\left[(s(x,Z) - s(x,y))^+\right] > 2c$. By supermodularity, for $x' > x$,

$$s\left(x', Z\right) - s\left(x', y\right) > s(x,Z) - s(x,y)$$

thus $E_Z\left[(s(x',Z) - s(x',y))^+\right] > 2c$, and $y \notin A(x')$. This being true for all $y < \underline{T}(x)$, this implies that $y < \underline{T}(x')$ as y cannot be greater than $\overline{T}(x')$. Hence $\underline{T}$ is nondecreasing. The same argument can be used to show that $\overline{T}$ is nondecreasing as well. ■

Equations (1.9) and (1.10) define implicitely functions $U_0(x)$ and $V_0(y)$. They are called Constant Surplus Conditions in the terminology of Atakan; they express indifference between waiting for one more period and incurring the search cost.

In particular, one has by derivation w.r.t. x

$$\mathbb{E}_Y\left[1\left\{Y \in A(x)\right\}\partial_x \Phi(x,Y)\right] = \mathbb{E}_Y\left[1\left\{Y \in A(x)\right\} U_0'(x)\right]$$

thus

$$U_0'(x) = \frac{\mathbb{E}_Y\left[1\left\{Y \in A(x)\right\}\partial_x \Phi(x,Y)\right]}{\mathbb{E}_Y\left[1\left\{Y \in A(x)\right\}\right]}$$

hence we recover Equation (1.8) from the Shimer-Smith model.

When $c \to 0$, Equations (1.9) and (1.10) imply that

$$U_0(x) + V_0(y) \geq \Phi(x,y)$$

and x and y match whenever equality holds.

Remark. In this model, agents have $-\infty$ reservation utility: they are forced to participate, and will continue to participate even if their $U_0(x)$ or $V_0(y)$ is negative (which may occur). Hence the first counterexample of Figure 1, with surplus function $\Phi(x, y) = (1 - x - y)^2$ cannot occur: at $x = y = 1/2$, agents may receive no output but their reservation utility is negative, so cost of waiting is too high, and they decide to match.

1.3 Identification

In order to estimate the model of Shimer and Smith, Jacquemet and Robin (2011) extend the model in several dimensions. They assume that there is a shock ε with cdf G on the joint production $\phi(x, y)$. x and y are willing to match whenever

$$\phi(x, y) + \sigma\varepsilon - u_0(x) - v_0(y) \geq 0$$

Hence instead of obtaining a set of acceptable matches A, one obtains a probability of matching of a pair (x, y) given by

$$\alpha(x, y) = 1 - G\left(-\frac{\phi(x, y) - u_0(x) - v_0(y)}{\sigma}\right) \quad (1.11)$$

(note that one recovers $\alpha \in \{0, 1\}$ when $\sigma \to 0$). A Search Equilibrium (SE) is now a quadruple $(u_0(x), v_0(y), \mu_0(x$
such that

$$u_0(x) = \theta \iint (\phi(x, y) + \varepsilon - u_0(x) - v_0(y))^+ \, d\mu_0(y) \, dG($$
$$v_0(y) = \theta \iint (\phi(x, y) + \varepsilon - u_0(x) - v_0(y))^+ \, d\mu_0(x) \, dG($$

and

$$\delta(p(x) - \mu_0(x)) = \rho\mu_0(x) \int \alpha(x, y) \, d\mu_0(y)$$
$$\delta(q(y) - \mu_0(y)) = \rho\mu_0(y) \int \alpha(x, y) \, d\mu_0(x).$$

In their model, λ is actually endogeneously determined as a function of the number of unmatched workers and firms

$$\lambda = \frac{M\left(\int d\mu_0\left(x\right), \int d\mu_0\left(y\right)\right)}{\int d\mu_0\left(x\right) . \int d\mu_0\left(y\right)}$$

and they use a Generalized Nash Bargaining solution, which implies that a part β of the surplus with respect to the status quo goes to x and a part $1-\beta$ goes to y (in the symmetric Nash sholtion, $\beta = 1/2$).

As before, one has

$$\mu\left(x, y\right) = \frac{\rho}{\delta}\alpha\left(x, y\right)\mu_0\left(x\right)\mu_0\left(y\right)$$

hence $\alpha\left(x, y\right)$ is identified from

$$\alpha\left(x, y\right) = \frac{\delta}{\rho}\frac{\mu\left(x, y\right)}{\mu_0\left(x\right)\mu_0\left(y\right)}$$

up to structural parameters ρ and δ.

Next, the payoffs $\frac{\phi(x,y)}{\sigma}$, $\frac{u_0(x)}{\sigma}$ and $\frac{v_0(y)}{\sigma}$ are actually identified up to structural parameters r, ρ and δ, β and G. Indeed, inverting equation (1.11) yields

$$\phi\left(x, y\right) - u_0\left(x\right) - v_0\left(y\right) = -\sigma G^{-1}\left(1 - \alpha\left(x, y\right)\right)$$

which alongs with the two set of equations

$$
\begin{aligned}
u_0(x) &= \theta \iint (\phi(x,y) + \varepsilon - u_0(x) - v_0(y))^+ \, d\mu_0(y) \, dG(\\
v_0(y) &= \theta \iint (\phi(x,y) + \varepsilon - u_0(x) - v_0(y))^+ \, d\mu_0(x) \, dG(
\end{aligned}
$$

yields identification of u_0, v_0, and ϕ. Hence we see that, in a very different framework from Choo and Siow, but in a similar spirit, observation of $\mu(x,y)$ has allowed us to identify the production function $\phi(x,y)$.

What is transfers (salaries) are observed? Let $w(x,y)$ be the average salary of worker x working for firm y. We set $\sigma = 1$ by scale invariance.As the sharing rules are well-defined, the equilibrium payoffs $u^{\varepsilon}(x,y)$ and $v^{\varepsilon}(x,y)$ such that

$$
u^{\varepsilon}(x,y) + v^{\varepsilon}(x,y) = \phi(x,y) + \varepsilon
$$

are given by

$$
\begin{aligned}
u^{\varepsilon}(x,y) &= u_0(x) + \beta(\phi(x,y) + \varepsilon - u_0(x) - v_0(y)) \\
v^{\varepsilon}(x,y) &= v_0(x) + (1-\beta)(\phi(x,y) + \varepsilon - u_0(x) - v_0(y))
\end{aligned}
$$

thus average share of surplus of matched partners are identified and given by

$$
\begin{aligned}
\bar{u}(x,y) &= E\left[u^{\varepsilon}(x,y)\,|\varepsilon \geq u_0(x)+v_0(y)-\phi(x,y)\right] \\
\bar{v}(x,y) &= E\left[v^{\varepsilon}(x,y)\,|\varepsilon \geq u_0(x)+v_0(y)-\phi(x,y)\right].
\end{aligned}
$$

Thus, as in the case previously discussed, the observation of transfers allow us to recoved the pre-transfer utilities and productivity $\alpha(x,y)$ and $\gamma(x,y)$ of workers and firms, by the formulas

$$
\begin{aligned}
\alpha(x,y) &= \bar{u}(x,y)-w(x,y) \\
\gamma(x,y) &= \bar{v}(x,y)+w(x,y)
\end{aligned}
$$

where $w(x,y)$ is the average wage of a worker x working for a firm y.

1.4 Empirical literature

Search frictions have the property to "blur" the sorting that might occur in the labor market. Search frictions create equilibrium mismatches such that similar workers will be matched to different firms and different firms will employ similar workers. As a result, we might expect that the sign and strength of the sorting will be difficult to obtain from data generated in an economy with search frictions.

Identifying the sign and the strength of the sorting has been of great interest in the economic literature since the early theoretical work by Koopmans and Beckman (1957) [**?**] and Becker (1973) [**?**]. The increasing availability of matched employer-employee panel data has opened new possibilities to test for the sign of sorting in the labor market. In their influential article, Abowd, Kramarz and Margolis (1999) [**?**] developed an empirical strategy to estimate firms and workers fixed effects in earnings regressions using large matched employer-employee panel data. This procedure consists of estimating the following wage equation

$$P_{it} = x_{it}\beta + \theta_i + \mathbf{\Psi}_{J(i,t)} + \varepsilon_{it} \tag{1.12}$$

where: P_{it} are the (log of) earnings of worker i at time t, x_{it} is a vector of time-varying covariates, θ_i reflects the worker's fixed effects and $\mathbf{\Psi}_{J(i,t)}$ is the fixed effect of firm J employing worker i at time t.

Abowd, Kramarz and Margolis naturally interpreted the sign and the magnitude of the correlation between firms and workers fixed effects as measures of the sign and strength of the sorting. At their surprise, when applying their estimation method to French and US data, they found that the correlation between θ_i and $\mathbf{\Psi}_{J(i,t)}$ was close to 0 or even negative in some cases while both sets correlated positively with measures of firms' productivity. This came at a surprise because as shown by Becker (1973) [**?**], in the canonical frictionless assignment model, as soon as the surplus of worker-firm pairs exhibits complementarities, the (rank-)correlation should be unity, i.e. positive assortative matching.

This puzzle has received some attention in the literature. The mainstream explanation is that the correlation between firms and workers fixed effects is simply not informative about the sorting in the economy. Among others, Lopes de Melo (2009) [**?**] argued that the firm fixed effect is indeed a poor measure of the productive capacity of the firm. As it turns out, $corr(\widehat{\theta}_i, \widehat{\mathbf{\Psi}}_{J(i,t)})$ is consistently downward biased when $\widehat{\theta}$ and $\widehat{\mathbf{\Psi}}$ are estimated using Abowd, Kramarz and Margolis's (1999) [**?**] methodology. This bias can be very large even if there is a large degree of sorting in the economy and since the reason for this bias lies in the economic mechanism of the model, it persists even as the sample size goes to infinity.

Applying the framework of Shimer and Smith (2000) [**?**] discussed earlier to the labor market, Lopes de Melo (2009) [**?**] generated simulated panel data allowing the estimation of Equation 1.12. Monte Carlo simulations showed that the alternative measure $corr\left(\widehat{\theta}_i, \frac{\sum_{j\in\widetilde{J}(i,t)}\widehat{\theta}_j}{\widetilde{N}_{\widetilde{J}(i,t)}}\right)$, where $\widetilde{J}(i,t)$ is the set of workers at firm $J(i,t)$ at t except worker i and $\widetilde{N}_{\widetilde{J}}$ is the number of coworkers of worker i at t, is a more reliable measure of sorting than $corr(\widehat{\theta}_i, \widehat{\mathbf{\Psi}}_{J(i,t)})$. However, the measure proposed by

Lopes de Melo (2009) [**?**] does not itself convey all the relevant information about the strength of sorting in the economy. As shown by Eeckhout and Kircher (2011) [**?**], this measure in fact captures only the range of wages each type of workers is willing to accept.

Why does the intuitively appealing measure $corr(\widehat{\theta}_i, \widehat{\mathbf{\Psi}}_{J(i,t)})$ not capture either the sign or strength of sorting in the economy? The intuition behind this result is that equilibrium wages in the Shimer and Smith (2000) [**?**] model, are monotonically increasing in the ability of workers but not in the productive capacity of the firm. In fact, the wages of a given worker have an inverted U-shape around the optimal match: clearly wages are low when matched with a bad firm but they are also low when matched with a good firm since this firm faces the opportunity cost of not matching with a better worker. This non-monotonicity of wages in the productive capacity of firms makes firm fixed effects in a wage regression a poor measure of productive capacities of firms.

1.5 References and notes

Section 1.1 follows Shimer and Smith (2000) [**?**], building on Sattinger (1995) [**?**], Lu and McAfee (1996) [**?**]. The reference for Section 1.2 is Atakan (2006) [**?**]. Section 1.3 relies on Jacquemet and Robin (2011) [**?**]. References for Section 1.4 are Abowd, Kramarz and Margolis (1999) [**?**], Lopes de Melo (2009) [**?**], and Eeckhout and Kircher (2011) [**?**].